Week 4
MATH 34B
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9. The acceleration (rate of change of velocity) of an object is 2t+1 meters per square second where t is the time in seconds. The velocity of the object at t=0 is 9 meters per second.
   (a) What is the velocity after t seconds?
   (b) When is the velocity 65 meters per second?
   (c) How far does the object move between t=0 and t=4?

a) $a(t) = 2t+1$

$v(t) = t^2 + t + c$

$v(0) = 9 \Rightarrow v(0) = 0^2 + 0 + c = 9 \Rightarrow c = 9$

$\Rightarrow v(t) = t^2 + t + 9$

b). $v(t) = 65$

$t^2 + t + 9 = 65$

$t^2 + t - 56 = 0$

$(t+8)(t-7) = 0$

$t = -8 \text{ or } 7$

Or can use quadratic formula!

So 7 sec, as -8 secs. makes no sense

c) distance moved $= \int_0^4 v(t)\,dt = \int_0^4 t^2 + t + 9\,dt$

$= \frac{t^3}{3} + \frac{t^2}{2} + 9t \Big|_0^4 = \frac{4^3}{3} + \frac{4^2}{2} + 9(4).$

14. The speed of car A after t minutes is $8t\ m/s$.

How long will it take the car to travel $\frac{100}{6}$ meters?

$v(t) = 8t.$

(position) $p(t) = \frac{8t^2}{2} + C = 4t^2 + C$

$C = 0$, because we're talking about distance travelled which means our starting position is 0, as at $t=0$, we've travelled nowhere.

$\Rightarrow p(t) = 4t^2$

set equal to $\frac{100}{6}$

$4t^2 = \frac{100}{6} \Rightarrow t = \sqrt{\frac{100}{24}}$

39. A tree trunk is approximated by a circular cylinder of height 100 meters and diameter 2 meters. The tree is growing taller at a rate of 4 meters per year and the diameter is increasing at a rate of 5 cm per year. The density of the wood is 1000 Kg per cubic meter.

How quickly is the mass of the tree increasing?

height $h = 100 + 4t.$

diameter $d = 4 + \frac{5}{100}t.$ Check last page.

$\Rightarrow r = \frac{4 + \frac{5}{100}t}{2} = 2 + \frac{5}{200}t.$

$V = 4\pi r^2 h = 4\pi\left(2 + \frac{5}{200}t\right)^2(100 + 4t)$

M = mass = density · volume = $1000 \cdot 4\pi\left(2 + \frac{5}{200}t\right)^2(100 + 4t)$

rate of change: $\frac{dM}{dt} = (1000 \cdot 4\pi)\left[2\left(2 + \frac{5}{200}t\right)(100 + 4t) \cdot \frac{5}{200} + 4\left(2 + \frac{5}{200}t\right)^2\right]$ Prod. rule.

We're talking about how quickly at this moment so set $t=0$.

$\frac{dM}{dt}(0) = 1000 \cdot 4\pi\left[2(2)(100)\left(\frac{5}{200}\right) + 4(2)^2\right]$

36. The population of a country Dnalgne is 100 million in 1997 and increasing at a rate of 0.6 million per year. The average annual income of a person in Dnalgne during 1997 was 24000 dollars per year and increasing at a rate of 500 dollars per year.

How quickly was the total income of the entire population rising in 1997?

population: $P = 100{,}000{,}000 + 600{,}000t$

avg. income: $C = 24{,}000 + 500t$.

$m =$ total income $= P \cdot C = (100{,}000{,}000 + 600{,}000t)(24{,}000 + 500t)$

$$\frac{dm}{dt} = 600{,}000(24{,}000 + 500t) \quad \text{(prod. rule)}.$$
$$+ (100{,}000{,}000 + 600{,}000t)(500).$$

set $t = 0$...

27. An artery has a circular cross section of radius 4 millimeters. The speed at which blood flows along the artery fluctuates as the heart beats. The speed after t seconds is $30 + 5\sin(2\pi t)$ meters per second. What volume of blood passes along the artery in one second?

In one second, $\int_0^1 30 + 5\sin(2\pi t)\,dt$ is how much distance the blood travelled. Since the cross section is circular, we can think of this distance as a "height" for a cylinder.

We evaluate $\int_0^1 30 + 5\sin(2\pi t)\,dt$

$$= 30t - 5\cos(2\pi t)\cdot\frac{1}{2\pi}\Big|_0^1$$
$$= 30 - 5\cos(2\pi) - (-5\cos(2\pi 0))$$
$$= 30 \text{ m} = 30{,}000 \text{ mm}.$$

So, $V = \pi(4)^2(30{,}000)$.

16. How quickly a leaf grows is proportional how big [ie the surface area] the leaf is. If the area of the leaf grows from $2cm^2$ to $3cm^2$ in 3 days, how long will it take for the leaf's area to increase to $5cm^2$?

$\frac{dA}{dt} \sim A \Rightarrow \frac{dA}{dt} = kA.$

$\int \frac{dA}{A} = \int k\,dt$

$\ln(A) = kt + C$

$A = e^{kt+C} = e^C e^{kt}.$

$A = Ce^{kt}$ (since e^C is constant anyhow) we'll call it C

know $C=2$, as that's how big it is at day 0.

$A = 2e^{kt}.$

At 3 days, it's 3 cm², so

$3 = 2e^{k\cdot 3}$

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$\frac{3}{2} = e^{3k}$

$\ln\frac{3}{2} = 3k.$

$k = \frac{1}{3}\ln(3/2).$

for time required to get 5cm²,

$5 = 2e^{kt}$, where k is as above, and solve for t...

50. Find a linear approximation to the function $f(x) = e^{x/500}$ for the range $0 < x < 100$. Do this by making the linear approximation equal to the function at the end points x=0 and x=100. Find the percent error in the approximation when (a) $x = 25$ and (b) $x = 50$.

slope: $\frac{e^{100/500} - e^{0/500}}{100} = \frac{e^{1/5} - 1}{100}$

have: point $(0, e^{0/500}) = (0,1)$

So, pt. slope tells us...

$y - 1 = \left(\frac{e^{1/5}-1}{100}\right)(x-0)$

so $y = \left(\frac{e^{1/5}-1}{100}\right)x + 1.$

a) % error is $\frac{\text{actual} - \text{estimate}}{\text{actual}} \cdot 100\%$

$= 100\% \cdot \frac{e^{25/500} - \left(\left(\frac{e^{1/5}-1}{100}\right)25 + 1\right)}{e^{25/500}}$

b) do same thing but replace 25 with 50...

39. $h = 100 + 4t$ (increases 4m per year)

$d = 2 + \frac{5}{100}t$ (increases 5 cm per year).

$\Rightarrow r = \dfrac{\left(2 + \frac{5}{100}t\right)}{2} = 1 + \frac{5}{200}t$

(volume) $V = \pi r^2 h = \pi\left(1 + \frac{5}{200}t\right)^2(100 + 4t)$

M (mass) = volume · density $= 1000\pi\left(1 + \frac{5}{200}t\right)^2(100 + 4t)$.

WANT TO MAXIMIZE M:

$\left(\frac{dM}{dt}\right) M' = 1000\pi\left[2\left(1 + \frac{5}{200}t\right)(100 + 4t)\frac{5}{200} + \left(1 + \frac{5}{200}t\right)^2 4\right]$ (prod. rule).

This is rate of change at year t afterwards.

But we're talking about now, so $t = 0$.

So plug $t = 0$ and get

$$\frac{dM}{dt} = 1000\pi\left[2(1)(100)\frac{5}{200} + (1)^2 4\right]$$